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Use of the exponentially weighted moving average control chart in industrial product control

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Abstract---The exponentially weighted moving average (EWMA) control chart stands out as a key element in modern quality control systems. and is a vital tool for enhancing process control and ensuring high standards of quality control through the effective integration of recent and past data via a weighting mechanism. It complements control charts for arithmetic means in terms of range, standard deviation, and combined standard deviation, in addition to moving average control charts, which are criticized for being memoryless and less sensitive in detecting small, continuous changes in the level of operations. In our study, we used the EWMA control chart using Minitab software, due to its high sensitivity to small shifts in the process mean, which allows for early detection and intervention. It is used to detect small changes, and this stage establishes stability in process outputs and adopts their outputs for long-term monitoring.

Keywords---exponentially weighted moving average, control chart, industrial product.

Introduction

Statistical process control is part of the field of statistical quality control and consists of several statistical methods and tools used to understand, control, monitor, and improve process performance. It is defined as a set of problem-solving tools used to stabilize the process and improve its capabilities by reducing differences and variations. Statistical process control is considered one of the greatest

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technological developments of the 20th century because it is based on sound fundamental principles, is easy to use, has a significant impact, and can be applied to any process. (Montgomery, 2012, p. 188)

Statistical control of production quality is a method of judging product quality and production process quality using probability theory and various statistical methods. It is applied to selected samples for the purpose of maintaining product quality on the one hand and intervening quickly to correct any errors on the other. The idea of statistical quality control is based on the fact that the units produced by a machine that a worker are rarely completely identical, as there must be some differences in a number of units due to a set of factors that interfere with the production process and lead to these differences (Jamal Taher Abu Al-Fateh Hijazi, 2002, p. 357).

1- Control charts

1-1 The concept of control charts

The idea of control charts goes back to Dr. **Walter A. Shewart,** who worked at Bell Laboratories in the US looking for reasons for poor phone quality. In 1924, Shewart developed a statistical chart to monitor product variables, which was the start of statistical quality control. The chart aims to understand and separate the sources of variation. Shewart is considered the first to distinguish between general causes of variation and special causes of variation. Shewart continued to develop his control chart theory until he published his famous book, Economic Control of Factory Produce, in 1931 (Ismail, 2006, p. 172).

A control chart is a graph that shows changes in product characteristics over time, so that through this chart, it is possible to distinguish between natural changes that are due to general causes inherent in the process and changes that are due to specific causes. The latter lead to product defects and operational errors, such as delays in delivery, which result in lower quality, increased costs, and ultimately customer dissatisfaction. (Aishuni, 2010, p. 79)

The horizontal axis of the map represents the sample numbers, known as partial groups, and the vertical axis represents the sample statistics (such as the arithmetic means of the samples). The sample statistics values for the partial groups are marked on the map in the form of points (or other symbols) connected by straight lines. Mathematically, the general model for a quality control chart (W) takes the following form: (Ismail, 2006, p. 172)

$$\begin{cases} UCL = \mu_w + L\sigma_w \\ CL \quad = \mu_w \\ LCL = \mu_w - L\sigma_w \end{cases}$$

Central Line (CL): represents the expected measurement average $\overline{X}$ $\bar{P}$ δ $\bar{R}$ Depending on the type of map used statistically, these values represent the average of the sample averages on which the measurement process is based.

Upper control limit (UCL): This is the maximum permissible level of the variable being measured. If its value exceeds this, it is considered a quality error that is not due to chance.

Lower control limit (LCL): This is the lowest permissible limit for the variable being measured without it being considered a quality error attributable to chance.

1-1 Types of control charts :

Control charts can be divided into two groups according to the type of data: Control charts for variables and control charts for attributes. The selection of the appropriate chart for use, in addition to the type of data, depends on the size of the sample, the sampling frequency, the quality attribute to be monitored, and the stage of chart application. Control charts can also be divided according to the type of data, application, and sample size, as summarized in the following table (Ismail, 2006, p. 178):

Table 1
Types of control charts

Variable	Property to be monitored	Subgroup	Map type
Variables	Process average	n≥1	Arithmetic mean, median, cumulative deviation sum map, exponential weighted arithmetic mean map
		n=1	Median, individual measurements, cumulative deviation sum map, exponentially weighted arithmetic mean map
Attributes	Process variation	n≥1	Range, Standard Deviation
		n=1	Moving Range
	Non-conformance ratio	n Constant variable	Map p
	Number of non-conforming units	n Constant	Map np
	Number of mismatches	n constant	Map c
	Number of mismatches	n constant that variable	Map u

1-2 Exponential weighted moving average (EWMA) map

The exponentially weighted moving average is a highly effective procedure for controlling the statistical adjustment of SPC processes, which has been used in industry for years. It is a procedure that has a number of design features that make it a highly desirable choice for selecting a control methodology. This type of chart has the following characteristics (Flaig, 2014, p. 21):

- It is very sensitive to small changes in the process, thus allowing the practitioner to detect changes early and respond to them.
- It is robust against abnormal data, meaning that it can be applied to distributions where the data is skewed or non-bell-shaped and will still provide accurate and reasonable results.

The exponentially weighted moving average (EWMA) control chart is used to quickly detect statistical deviations in the process and is good at detecting small changes in the mean value of the process. The EWMA chart is considered better than the range and mean control chart $\left(\overline{X}-R \text{ charts}\right)$ for early detection of statistical control deviations in the process. It is used in production processes as well as in service processes (financial and administrative in particular) where it is difficult to obtain samples containing several observations at a time. In this chart, only one observation (n=1) is used in each sample, and the value of this observation may be the average of a set of individual data, ratios and fractions, or individual measurements of a single variable (Aishuni, 2013, p. 235).

1- Practical steps for creating an exponentially weighted moving average map (Eishouni, 2014, p. 240) :

Step 1: We calculate the approximate value of the standard deviation of the process using one of the above-mentioned relationships, and we also calculate the mean value of the process from the collected data.

Step 2: We determine the value of the weighting coefficient (λ) so that it is $\left(0<\lambda<1\right)$

Step 3: We calculate the values of the weighted moving averages Z_t starting from time (t=1) and taking into account that $Z_0 = \mu$.

Step 4: We calculate the control limits on the chart according to the previous equations.

Step 5: Draw the control chart by plotting the values of Z_t as a function of the sample number or time (t) with the center line $Z_0 = \mu$ as the exact limits.

Step 6: Analyze the control chart to determine whether the process is statistically controlled or not.
2- Types of Exponential Weighted Moving Average (EWMA) maps.
The exponential weighted moving average (EWMA) map can be divided into two basic types:
- Exponential weighted moving average map for partial groups.
- Exponential weighted moving average map for individual groups.

1.1 Exponential weighted moving average map for partial groups:

The exponentially weighted moving average ($\left(Z_i\right)$) for partial group number (i) takes the following form (Saccucci, 1990, p. 11) :

$$Z_i = \lambda \overline{X}_i + \left(1-\lambda\right) Z_{i-1} \text{ ....(01)} \qquad \textbf{for i>0}$$

Where: $\left(0<\lambda<1\right)$

λ : Weighting coefficient for the current observation. In general, this coefficient is between 0 and 1.

$\overline{X}_i$: The arithmetic means of subset number i.

The total mean $\left(\overline{\overline{X}}\right)$ or the target value (T) is used for the starting point $\left(Z_0\right)$.
Equation (01) can be rewritten as follows:

$$Z_i = \lambda \overline{x_i} + \left(1-\lambda\right) Z_{i-1} = Z_{i-1} + \lambda\left(\overline{x_i} - Z_{i-1}\right)$$

$$= \lambda \overline{x_i} + \left(1-\lambda\right)\left[\lambda \overline{x_i} + \left(1-\lambda\right) Z_{i-2}\right]$$

$$= \lambda \overline{x_i} + \left(1-\lambda\right)\lambda \overline{x}_{i-1} + \left(1-\lambda\right)^2 Z_{i-2}$$

$$= \lambda \overline{x_i} + \left(1-\lambda\right)\lambda \overline{x}_{i-1} + \left(1-\lambda\right)^2\left[\lambda \overline{x}_{i-2} + \left(1-\lambda\right) Z_{i-3}\right]$$

$$= \lambda \overline{x_i} + \left(1-\lambda\right)\lambda \overline{x}_{i-1} + \left(1-\lambda\right)^2 \lambda \overline{x}_{i-2} + \left(1-\lambda\right)^3 Z_{i-3}$$

Continuing to substitute the values of Z_{i-j} where j=3,4,5……..

$$Z_i = \lambda \sum_{j=0}^{i-1} \left(1-\lambda\right)^j \overline{X}_{i-j} + \left(1-\lambda\right)^i Z_0 \text{............(02)}$$

It is clear from equation (02) that any point on the map is the weighted mean of the partial sums preceding it. It should be noted that the weights $\lambda\left(1-\lambda\right)^j$ decrease exponentially due to the change in the value of the exponent j. hence the name exponential weighted arithmetic mean. It should also be

noted that the weighting constant (λ) determines the effect of the previous points on the current point. It can be concluded from equations (01) and (02) if the value of the weighting constant approaches one, this means that greater weight is given to the current partial group average and the map approaches the arithmetic mean map. On the other hand, if the value of the weighting constant is closer to zero, this means that less weight is given to the current partial group average. (Ismail, 2006, p. 266)
Variance of the exponential moving average of the partial groups**:** If the averages of the partial groups $(\overline{X}_i)$ are independent random variables with variance $\left(\frac{\sigma^2}{n}\right)$, then the variance Z_i can be calculated as follows (Ismail, 2006, p. 268) :

$$V(Z_i) = V\left[\lambda \overline{x}_i + (1-\lambda) Z_{i-1}\right] = \lambda^2 V(\overline{x}_i) + (1-\lambda)^2 V(Z_{i-1})$$

Given the proximity of the values of consecutive observations, it can be assumed that the variance $V(Z_i)$ is approximately equal to the variance $V(Z_{i-1})$. The equation can then be rewritten as follows:

$$V(Z_i)\left[1-(1-\lambda)^2\right] = \lambda^2 \sigma^2 / n$$

Therefore, the variance of Z_i is

$$V(Z_i) = \frac{\lambda}{2-\lambda}\left(\frac{\sigma^2}{n}\right)$$

For small values of i, the variance of $Z_1$is derived as follows:

$$V(Z_1) = \lambda^2 V(\overline{x}_i) + (1-\lambda)^2 V(Z_0) = \lambda^2\left(\frac{\sigma^2}{n}\right)$$

Variance Z_2

$$V(Z_2) = \lambda^2\left(\frac{\sigma^2}{n}\right) + (1-\lambda)^2 \lambda^2\left(\frac{\sigma^2}{n}\right) = \left(\frac{\sigma^2}{n}\right)\lambda^2\left[1+(1-\lambda)^2\right]$$

In the same way, the variance of Z_3 and Z_4 is derived as follows:

$$V(Z_3) = \left(\frac{\sigma^2}{n}\right)\lambda^2\left[1+(1-\lambda)^2+1+(1-\lambda)^4\right]$$

$$V(Z_4) = \left(\frac{\sigma^2}{n}\right)\lambda^2\left[1+(1-\lambda)^2+1+(1-\lambda)^4+(1-\lambda)^6\right]$$

In general, the variance Z_i takes the following form:

$$V(Z_i)=\left(\frac{\sigma^2}{n}\right)\lambda^2\left[1+(1-\lambda)^2+1+(1-\lambda)^4+(1-\lambda)^6+\ldots+(1-\lambda)^{i-1}\right]$$

$$=\left(\frac{\sigma^2}{n}\right)\lambda^2\left[\frac{1-(1-\lambda)^{2i}}{1-(1-\lambda)^2}\right]$$

$$=\left(\frac{\sigma^2}{n}\right)\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

Control limits for the exponential moving average map of partial groups: To calculate the control limits for the map, we use the following equations:

$$LCL_i=Z_0-L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}$$

Minimum limit for the control map:

Center line: μ $Z_0=$

$$UCL_i=Z_0+L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}$$

Upper limit of the control chart:

Z_0: Total mean $\overline{\overline{X}}$ or target value

L: Width of the control limits, usually (03)

σ: Standard deviation of X_i, estimated using either

$\sigma=\frac{\overline{R}}{d_2}$ or $\sigma=\frac{\overline{S}}{C_4}$

n: Size of the partial group

From the two previous equations, we note that the limit $\left[1-(1-\lambda)^{2i}\right]$ approaches or equals one as the value of i increases, i.e., when the period from which the data is taken increases. Therefore, the variance Z_i can be used according to the previous formula to calculate the control limits, which in this case are calculated as follows (Rashid, 2017, p. 89) :

$$UCL=Z_0+L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$

Upper limit:

$$LCL=Z_0-L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$

Lower limit:

However, the first equations remain more accurate for calculation.

The sensitivity of the exponentially weighted moving average map to detect changes in process outputs depends on the values of (L) and the weighting constant (λ). As for the value of (L), it is preferable that it be equal to (03), especially with large weighting constant values. As for determining the value of the weighting constant, the best results are achieved when the constant value is between (0.1) and (0.3) according to Farnum (1994) and between (0.05) and (0.25) According to Montgomery, and in general, Montgomery suggests using small values for the weighting constant to detect small changes. Lucas and Saccucci (1990) prepared tables to help select the appropriate weighting constant value (Ismail, 2006, pp. 269-270).

2- Exponential moving average map for individual observations:

It should be noted that the above equations apply to the exponentially weighted moving average map in the case of sub-groups larger than one observation. In the case of individual observations, the same equations are used, replacing$\overline{x_i}$ withx_i and replacing the variance Z_i for partial groups with the variance of individual views. The equations for the exponentially weighted moving average and the observation index can be rewritten as follows:

Z_i for view count i is (Ismail, 2006, p. 271):

$$Z_i = \lambda \overline{X}i + (1-\lambda) Z_{i-1} \;..........(01) \qquad \textbf{for i>0}$$

Minimum monitoring limit:

$$LCL = Z_0 - L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$

Upper monitoring limit:

$$UCL = Z_0 + L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$

Applied study:

In order to perform statistical quality control of a specific industrial product manufactured by a company using the exponential moving average map for partial groups, we recorded measurements for one of the institution's industrial products for a period of 25 days and, using simple random sampling, took partial groups. The results are recorded in Table 2.

Table 2
Recorded measurements for the product under study

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1	1025	1025	1037	1044	1024	1040	1037	1037	1037	1036	1040	1037
2	1026	1026	1028	1024	1025	1026	1025	1023	1025	1022	1024	1027
3	1032	1027	1032	1030	1030	1033	1038	1028	1021	1030	1028	1033
4	1028	1031	1034	1036	1037	1035	1031	1035	1039	1033	1035	1039
5	1022	1020	1025	1027	1025	1020	1022	1020	1024	1020	1026	1020
6	1042	1041	1037	1041	1037	1039	1040	1038	1037	1038	1036	1041
7	1032	1030	1030	1027	1025	1029	1034	1030	1027	1030	1030	1030
8	1027	1038	1030	1033	1031	1047	1034	1051	1036	1037	1031	1043
9	1028	1036	1032	1045	1033	1032	1031	1029	1034	1038	1031	1049
10	1048	1025	1030	1038	1047	1034	1028	1018	1027	1034	1037	1026
11	1026	1037	1022	1030	1034	1039	1028	1031	1037	1025	1039	1020
12	1035	1037	1036	1046	1037	1033	1036	1032	1039	1031	1038	1029
13	1031	1044	1036	1029	1034	1034	1032	1029	1033	1039	1029	1034
14	1032	1034	1031	1032	1037	1027	1029	1035	1030	1030	1029	1033
15	1030	1033	1031	1038	1042	1022	1045	1027	1026	1022	1026	1027
16	1025	1029	1037	1035	1033	1035	1031	1029	1033	1034	1035	1032
17	1034	1022	1020	1035	1044	1037	1030	1024	1032	1023	1040	1029
18	1022	1033	1027	1034	1036	1027	1038	1025	1042	1027	1031	1026
19	1039	1030	1033	1022	1020	1037	1022	1029	1024	1045	1040	1028
20	1033	1032	1035	1037	1037	1032	1037	1033	1030	1033	1024	1033
21	1028	1032	1032	1032	1037	1039	1035	1033	1032	1034	1031	1029
22	1040	1034	1035	1033	1028	1030	1026	1021	1032	1031	1029	1024
23	1033	1037	1037	1040	1027	1028	1036	1040	1032	1037	1031	1041
24	1038	1032	1036	1041	1038	1035	1037	1039	1037	1034	1035	1038
25	1032	1035	1036	1038	1032	1035	1039	1035	1036	1040	1035	1038

Exponential moving average map of partial groups according to Montgomery with a weighting constant of $(\lambda = 0.25)$:

In this case, we will extract the control chart by determining the weighting constant value for the $(\lambda = 0.25)$ values to see if the chart is statistically stable or not.

1- In this case, we extract the control limits using the standard deviation in terms of the mean range $\sigma = \bar{R}/d_2$ and the standard deviation in terms of the mean standard deviation $\sigma = \bar{S}/C_4$. Using the statistical program Minitab, we obtain the control limits.

1- **Using the standard deviation estimate based on the mean range (** $\sigma = \bar{R}/d_2$ **),** the control limits are

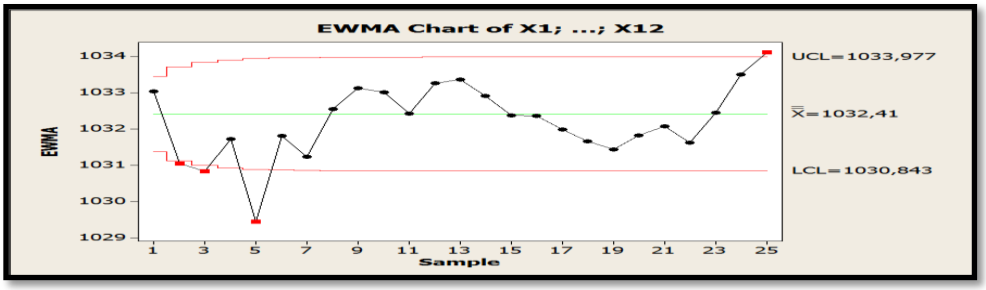

Figure 1. Map of the exponentially weighted moving average for partial groups

Interpretation: From Figure (01), it can be seen that all points related to the map of the exponentially weighted moving average of the partial groups according to Montgomeri and the standard deviation estimated by the mean range fall within the upper and lower control limits, except for samples 02, 03, 05, and 25, which fell outside the control limits. Therefore, the process is not statistically stable. By excluding these samples that are outside the control limits, we will have a new general mean $\left(\bar{\bar{X}}_{new}\right)$ and a new range mean $\left(\bar{R}_{new}\right)$ as follows:

$$\begin{cases} \bar{\bar{X}}_{new} = \dfrac{21696,4}{21} = 1033,16 \\ \bar{R}_{new} = \dfrac{352}{21} = 16,76 \end{cases}$$

We then recalculate the standard deviation as follows:

$$\sigma = \bar{R}/d_2 \Rightarrow \sigma = 16,76/03,258 = 05,14$$

The new control chart after excluding samples 02, 03, 05, and 25 is as follows:

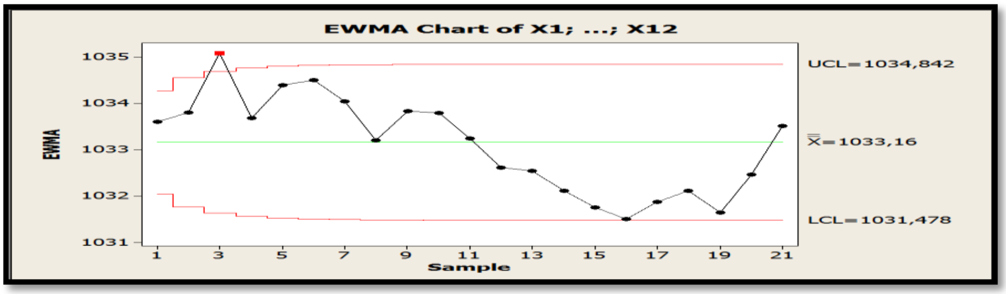

Figure 2: Exponentially weighted moving average map for partial groups with a weighting constant $(\lambda = 0,25)$ and estimated standard deviation $\sigma = \bar{R}/d_2$

Interpretation: Figure 2 shows the exponentially weighted moving average map of the subgroups according to Montgomeri and the standard deviation estimated by the mean range. There is a point outside the control limits. The process is statistically unstable, and the previous control chart shows that the mean outside the control limits is represented by sample No. 03. By excluding these samples that are outside the control limits, we will have a new overall mean $\left(\bar{\bar{X}}_{new}\right)$ and a new range mean $\left(\bar{R}_{new}\right)$ as follows:

$$\begin{cases} \bar{\bar{X}}_{new} = \dfrac{20657,5}{20} = 1032,875 \\ \bar{R}_{new} = \dfrac{346}{20} = 17,30 \end{cases}$$

Then, we recalculate the standard deviation as follows:

$$\sigma = \bar{R}/d_2 \Rightarrow \sigma = 17,30/03,258 = 05,31$$

The new control chart after excluding sample No. 03 is as follows:

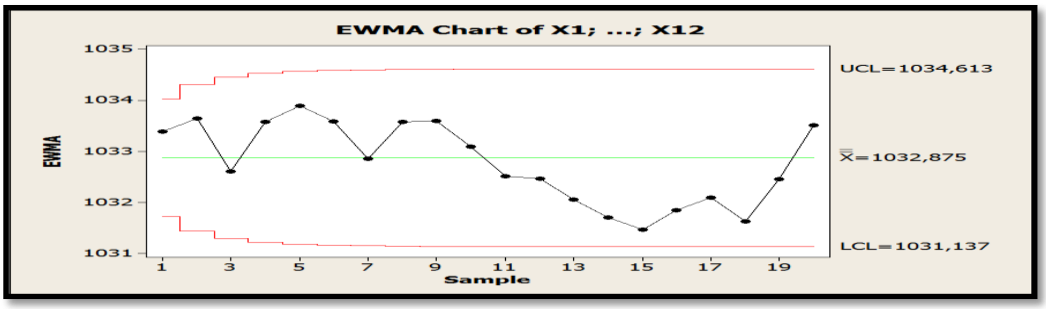

Figure (03): Exponentially weighted moving average map for partial groups with a weighting constant $(\lambda = 0,25)$ and estimated by standard deviation $\sigma = \bar{R}/d_2$

Interpretation: From Figure (03), it can be seen that all points related to the exponential weighted moving average map of the sub-groups according to Montgomeri and with the standard deviation estimated by the mean range fall within the upper and lower control limits. Therefore, the process is statistically stable. The control limits can be adopted in the long term to monitor the process in the future using a single method of data collection and the size of the subgroups, taking into account the revision of the control limits in the event of a change in measurements.

1-3 Using the standard deviation estimate in terms of the mean range $\sigma = \bar{S}/C_4$ **:** The control limits are

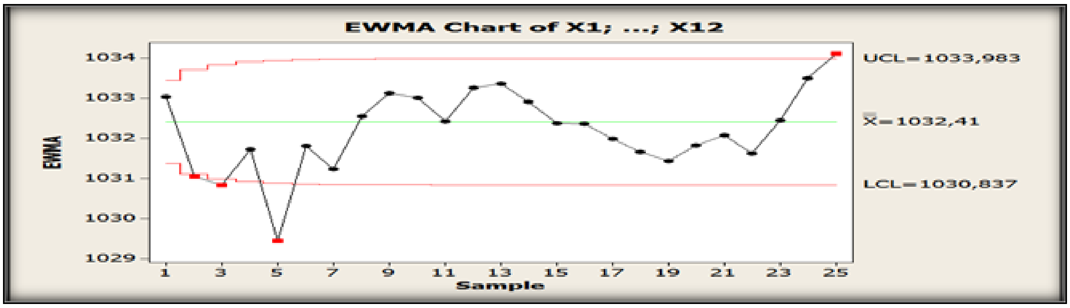

Figure (04): Map of the exponentially weighted moving average of sub-groups with a weighting constant $(\lambda = 0,25)$ **and estimated by standard deviation** $\sigma = \bar{S}/C_4$

Interpretation: From Figure (04), it can be seen that all points related to the map of the exponentially weighted moving average of the partial groups according to Montgomeri and the standard deviation estimated by the mean standard deviation fall within the upper and lower control limits, except for samples 02, 03, 05, and 25, which fell outside the control limits. Therefore, the process is not statistically stable. Excluding these samples that are outside the control limits, we will have a new general mean $\left(\bar{\bar{X}}_{new}\right)$ and a new range mean $\left(\bar{S}_{new}\right)$ as follows:

$$\begin{cases} \bar{\bar{X}}_{new} = \dfrac{21696,4}{21} = 1033,16 \\ \bar{S}_{new} = \dfrac{106,554}{21} = 05,074 \end{cases}$$

We then recalculate the standard deviation as follows:

$$\sigma = \bar{S}/C_4 \Rightarrow \sigma = 05,074/0,9776 = 05,19$$

The new observation map after excluding samples 02, 03, 05, and 25 is as follows:

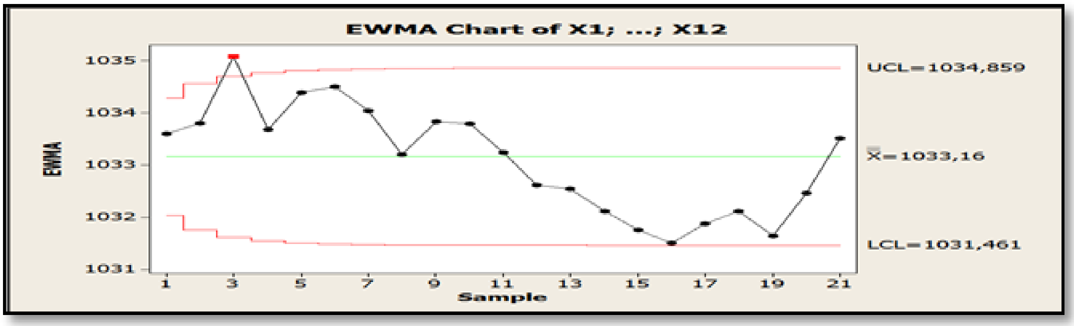

Figure 5: Exponentially weighted moving average map for partial groups with a weighting constant $(\lambda = 0,25)$ and estimated standard deviation $\sigma = \bar{S}/C_4$ after excluding samples 02, 03, 05, and 25.

Interpretation: From Figure 5, it can be seen that all points related to the exponentially weighted moving average map of the partial groups according to Montgomeri and the standard deviation estimated by the mean standard deviation fall within the upper and lower control limits, except for sample No. 03, which fell outside the control limits. Therefore, the process is not statistically stable. By excluding these samples that are outside the control limits, we will have a new overall mean $\left(\bar{\bar{X}}_{new}\right)$ and a new standard deviation $\left(\bar{S}_{new}\right)$, as follows:

$$\begin{cases} \bar{\bar{X}}_{new} = \dfrac{20657,5}{20} = 1032,875 \\ \bar{S}_{new} = \dfrac{104,533}{20} = 05,23 \end{cases}$$

Then we recalculate the standard deviation as follows:

$$\sigma = \bar{S}/C_4 \Rightarrow \sigma = 05,23/0,9776 = 05,35$$

The new control chart after excluding sample No. 03 is as follows:

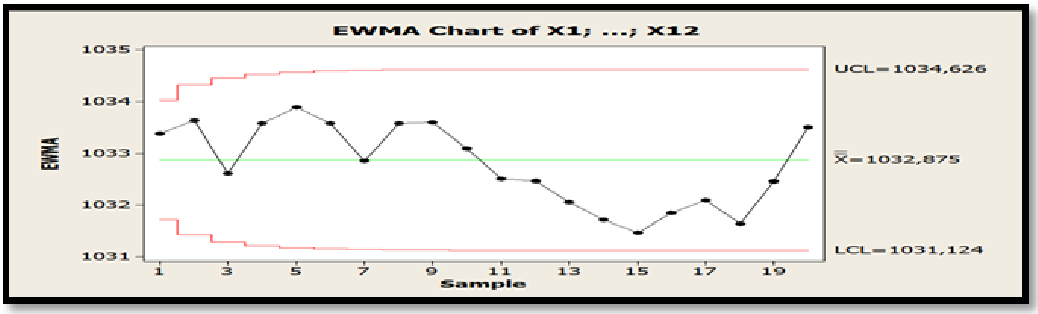

Figure 6: Exponentially weighted moving average map for partial groups with a weighting constant $(\lambda = 0,25)$ and estimated standard deviation $\sigma = \bar{S}/C_4$ after excluding sample No. 03

Interpretation: From Figure (06), all points using the exponential moving average map of partial groups according to Montgomeri fall within the upper and lower control limits, and therefore the process is statistically stable. The control limits for this chart can be adopted in the long term to monitor the process in the future using a single method of data collection and the size of the subgroups, taking into account the review of the control limits in the event of a change in measurements, and reviewing the records to address the causes of quality loss on days when the samples are out of control.

Through the practical application of the exponentially weighted moving average (EWMA) chart, the study demonstrated its high effectiveness in assessing process stability and early detection of small deviations in the average output, and from the figures extracted using MINITAB software and when using a weighting coefficient ($\lambda = 0.25$) that enhances the sensitivity of the chart to minor changes.
The first figures showed that the process was statistically unstable, as several points appeared outside the control limits (e.g., samples: 02, 03, 05, 09, 15, 19). This was a clear indication that there were specific reasons affecting the performance of the process, which required the exclusion of those points and the re-estimation of the process parameters (mean and standard deviation). This procedure resulted in more accurate new control limits, as reflected in the subsequent figures.

The improved charts after the gradual exclusion of points outside the control limits also showed that the process was stabilizing, which was clearly evident in the final stage, where the charts were free of any points exceeding the control limits. This is evidence of the effectiveness of the EWMA method in isolating specific causes of variation and returning the process to a state of statistical control.

The results obtained from all figures confirm that the EWMA chart is not just a tool, but an integrated approach to statistical control, capable of supporting decision-making and improving the quality of processes in the long term. Its importance is particularly evident in production and service environments characterized by gradual changes or when it is difficult to obtain large partial sets.

Based on this, the study recommends adopting the EWMA chart as a standard tool for process control, with the need to periodically update control limits, review points outside control limits in a timely manner, and analyze their causes in order to maintain process stability and product quality sustainability.

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